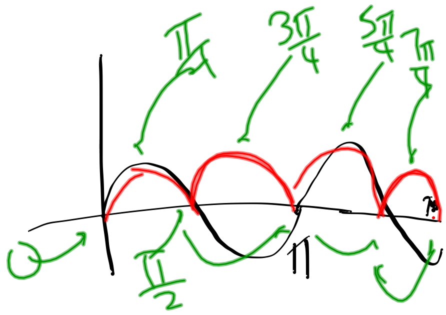
5.2/39

$y = |\sin(2x)|$



period of $\sin(2x)$ is π

$0 \longrightarrow 2\pi$

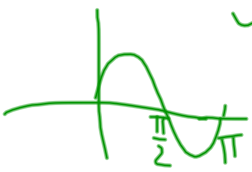
$2x = 0$ $\quad$ $2x = 2\pi$

$x = 0$ $\quad$ $x = \pi$

abs value $\Rightarrow \frac{\pi}{2}$

$y = |\sin(2x)|$

$$y = \begin{cases} \sin(2x) & \text{when } \sin(2x) \geq 0 \quad x \in [0, \frac{\pi}{2}], [\pi, \frac{3\pi}{2}] \\ -\sin(2x) & \text{when } \sin(2x) < 0 \quad x \in (\frac{\pi}{2}, \pi) \text{ and } (\frac{3\pi}{2}, 2\pi) \end{cases}$$



$$y' = \begin{cases} 2\cos(2x) & x \in (0, \frac{\pi}{2}) \text{ and } (\frac{3\pi}{2}, 2\pi) \\ -2\cos(2x) & x \in (\frac{\pi}{2}, \pi) \text{ and } (\frac{3\pi}{2}, 2\pi) \end{cases}$$

$\frac{\pi}{4}, \frac{3\pi}{4}, \ldots \Rightarrow y' = 0$

5.2/34

$$y = \frac{x}{x+2}$$

$$y' = \frac{(1)(x+2) - (x)(1)}{(x+2)^2}$$

$$= \frac{2}{(x+2)^2}$$

$f'(x)$

- inc
- dec
- rel max
- rel min
- critical #s

id "cut points" = critical #s

$y' = 0$

none

$2 \neq 0$

y' und $\Rightarrow (x+2)^2 = 0$

$x = -2$

sign chart of f'

$+ + + \mid + + +$

-2

each HALF fn ($x < -2, x > -2$)

always increases, so No rel extrema

1) $y' = \frac{2}{(x+2)^2}$

$= 2(x+2)^{-2}$

$y'' = 2\left(-2(x+2)^{-3}(1)\right)$ $\frac{d}{dx}(x+2)$

$f''(x)$

concavity
- up
- down

point of inflection

* 2ND derivative test

+ + + | − − − −
−2

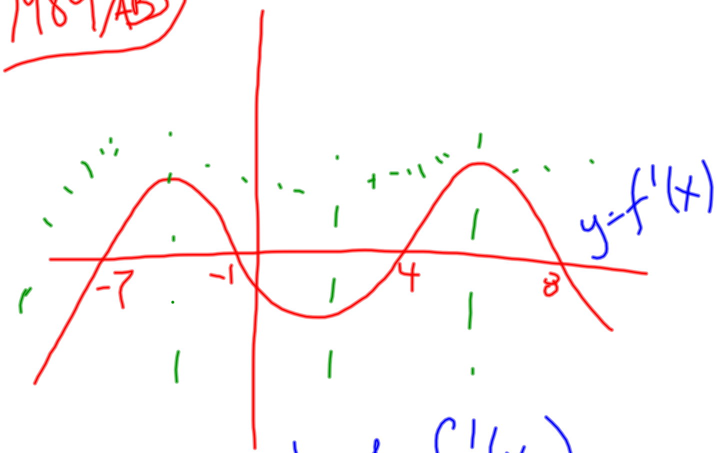
2)

$y' \frac{2}{(x+2)^2}$

$y'' = \frac{(0)(x+2)^2 - (2)(2(x+2)(1))}{[(x+2)^2]^2} = \frac{-2(x+2)}{(x+2)^4}$

$= \frac{-2}{(x+2)^3}$

1989/AB5



sign chart of $f'(x)$

- - | + + | - - - | + + | - -
-7 -1 4 8

sign chart of $f''(x)$

+ + + + + | - - - - - | + + + + | - - -
-3 2 6

Concave down is indicated by

* negative second derivative

[or]

* decreasing first derivative

a) $x = -7, -1, 4, 8$ — +1 pt for each

b) rel max is when f' chgs fr. pos to neg at crit # — +

$x = -1$ +1

$x = 8$ +1

c) $(-3, 2)$ +1

and

$(6, 10)$ +1

5.2/24 $f(x) = \frac{1}{2}x - \sin x \; ; 0 < x < 2\pi$

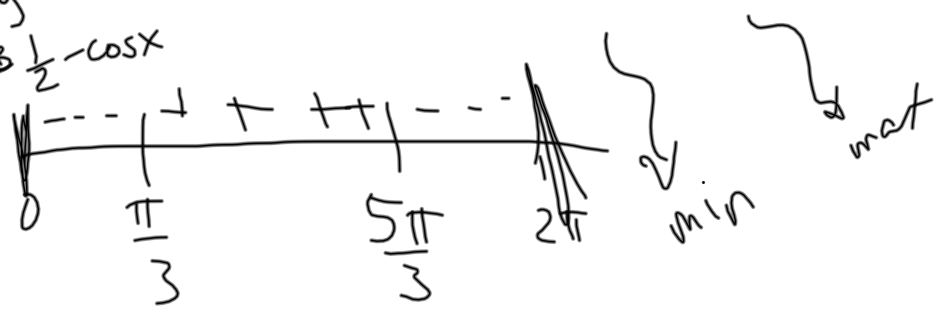
$f'(x) = \frac{1}{2} - \cos x$

f' always defined

$f' = 0$ when $\frac{1}{2} - \cos x = 0$

$\frac{1}{2} = \cos x \; ; \; x = \frac{\pi}{3}, \frac{5\pi}{3}$

s.c of $\frac{1}{2} - \cos x$



$f'(x) = \frac{1}{2} - \cos(x)$

$f''(x) = \sin(x)$

$f''\left(\frac{\pi}{3}\right) = \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2} > 0$

$\therefore x = \frac{\pi}{3}$ is a rel min

$f''\left(\frac{5\pi}{3}\right) = \sin\left(\frac{5\pi}{3}\right) = \frac{-\sqrt{3}}{2} < 0 \; \therefore x = \frac{5\pi}{3}$ is a rel max

5.2/36

$$f(x) = x^2 e^x$$

$$f'(x) = 2xe^x + x^2e^x =$$

$$xe^x(2+x) = 0$$

und? NO

$= 0$ yes

$$x = 0, -2$$

$xe^x(2+x)$

$+ + + + \;|\; - - - \;|\; + + + +$

$-5 \quad -2 \quad -1 \quad 0 \quad 1$

relative

MIN, $x=0$

MAX: $x=-2$

5.2/35) $f(x) = \ln(1+x^2)$

$$f'(x) = \frac{1}{1+x^2}(2x)$$

$$= \frac{2x}{1+x^2}$$

find crit #s

$f'(x)$ und? No $x^2 \neq -1$

$f'(x) = 0$

$2x = 0$

$x = 0$

chart sign of f'

$- - - | + + + + +$ (at 0)

$\therefore x = 0$ is where rel MIN occurs

first derivative: slope, increasing/decreasing, critical #s, rel extrema [first derivative]

$$f'(x) = \frac{2x}{1+x^2}$$

$$f''(x) = \frac{(2)(1+x^2) - (2x)(2x)}{(1+x^2)^2}$$

$$= \frac{2 + 2x^2 - 4x^2}{(1+x^2)^2}$$

$$= \frac{2 - 2x^2}{(1+x^2)^2}$$

second derivative: [2^{ND} deriv. test], concave up, concave down, inflection pts

f'' und? NEIN

$2 - 2x^2 = 0$

JA

$2(1-x)(1+x) = 0$

$\Rightarrow x = -1, +1$

$- - | + + + | - -$ (at -1 and $+1$)

$f''(0) = +$ ... rel MIN

5.2/39

$f(x) = |\sin(2x)|$

what is period of $\sin(2x)$?

π

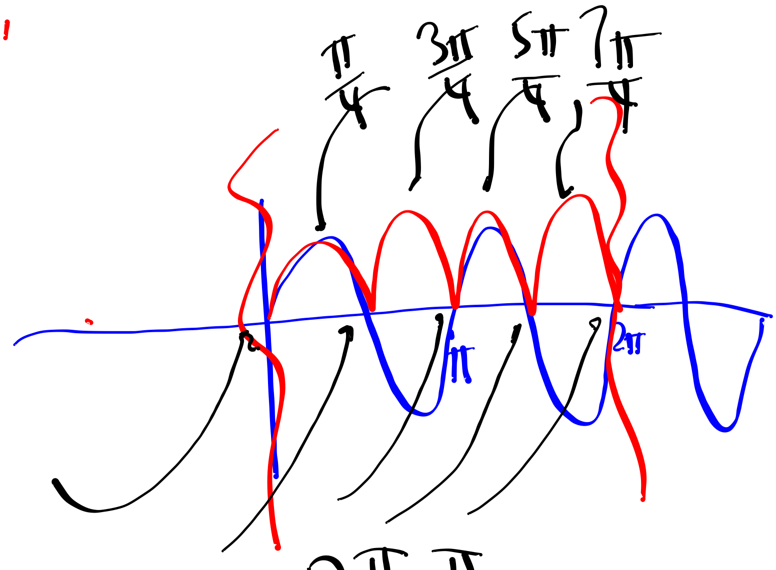
0 —————— 2π

$2x = 0$? $2x = 2\pi$?

$x = 0$ —————— $x = \pi$

what is period of $|\sin(2x)|$?

$\frac{\pi}{2}$

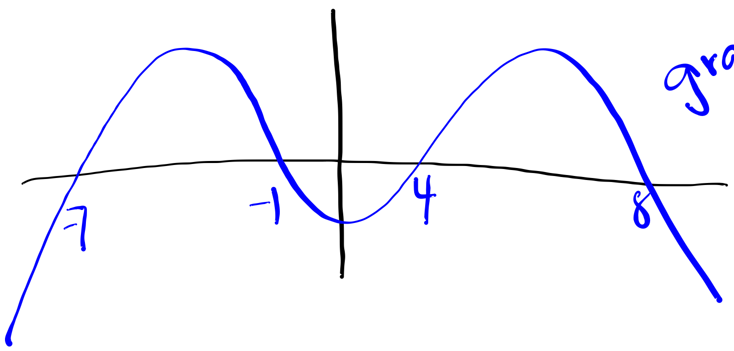


$0, \frac{\pi}{2}, \pi, \ldots$

$$y = |\sin(2x)|$$

$$y = \begin{cases} \sin(2x), & \text{when } \sin(2x) \geq 0 \\ & [0, \tfrac{\pi}{2}] \text{ and } [\pi, \tfrac{3\pi}{2}] \\ -\sin(2x), & \text{when } \sin(2x) < 0 \\ & (\tfrac{\pi}{2}, \pi) \cup (\tfrac{3\pi}{2}, 2\pi) \end{cases}$$

$$y' = \begin{cases} 2\cos(2x), & x \in (0, \tfrac{\pi}{2}) \cup (\pi, \tfrac{3\pi}{2}) \\ x \to \tfrac{\pi}{2}^- \quad 2\cos(\pi) = -2 & \\ -2\cos(2x), & x \in (\tfrac{\pi}{2}, \pi) \cup (\tfrac{3\pi}{2}, 2\pi) \\ x \to \tfrac{\pi}{2}^+ \quad -2\cos(\pi) = +2 & \end{cases}$$



graph of $f'(x)$

sign chart of $f'(x)$

$- \; - \;|\; + \; + \;|\; - \; - \;|\; + \; + \;|\; - \; -$

$-7 \qquad -1 \qquad 4 \qquad 8$

a) $x = -7, -1, 4, 8$ } +1 each

(b) rel MAXes happen when f' goes from positive to negative +1

$x = -1, 8$ } +1 each

(c)

s.c. of $f''(x)$

$+ + + \;|\; - - - \;|\; + + + \;|\; - - -$

$-3 \qquad 2 \qquad 6$

$(-3, 2)$ and $(6, 10)$